Dynamic Modelling

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A dynamic model provides information about why a robot moves, we can express a dynamic model with the following equation:

$\tau = M(q)\ddot{q} + b(q,\dot{q})\dot{q} + g(q)$

Where:

τ: vector of joint torques (revolute joints) or forces (prismatic joint)

q: vector of joint position

$M(q)$: generalized mass matrix

$b(q,\dot{q})$: coriolis and centrifugal terms

$g(q)$: gravitational terms

The above equation is the inverse dynamic model which receives the joint positions, velocities and accelerations to determine the joint torques. It can be based on Newtonian and/or Lagrangian mechanics formulation.

Langrage Formulation

This formulation describes the behavior of any dynamic system in terms of work and energy stored in the
system. The equations for the joint $1 \cdots n$

$$\tau_i = \frac{d}{dt}\frac{\partial L}{\partial q_i} - \frac{\partial L}{\partial q_i}$$

Where:

L: *Langragian of the robot*

T: *Kinetic Energy*

U: *Potential Energy*

$L = T - U$

The kinetic energy can be expressed as:

$$T(q,\dot{q}) = \frac{1}{2}\dot{q}^T\left(\sum_{i=1}^{n_b}\left(J_{S\,i}^T m_i J_{S_i} + J_{R\,i}^T \theta_{S_i} J_{R_i}\right)\right)\dot{q}$$

$$M(q) = \sum_{i=1}^{n_b}\left(J_{S\,i}^T m_i J_{S_i} + J_{R\,i}^T \theta_{S_i} J_{R_i}\right)$$

Where:

m_i: *The mass*

J_{S_i}: *Geometric Jacobian defined by the kinematics*

θ_{S_i}: *Inertia matrix*

J_{R_i}: *Rotation Jacobian defined by the kinematics*

The gravitational potential energy is expressed as:

$F_{g_i} = m_i g I_g^e$

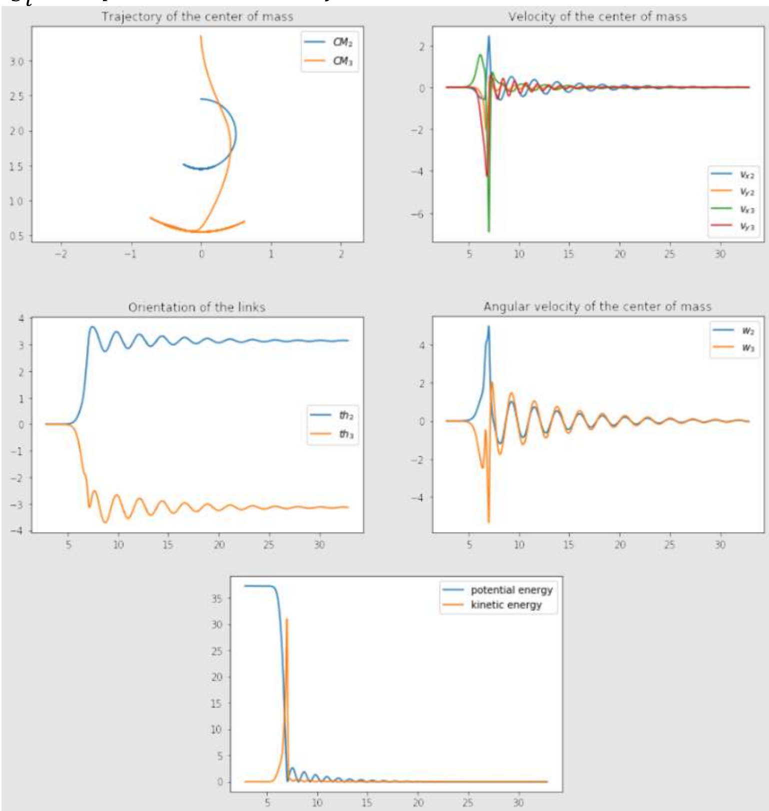
Where:

I_g^e: *Unit vector acting throught the center of mass*

$$U_g = -\sum_{i=1}^{n_b} r_{S\,i}^T F_{g_i}$$

Where:

r_{S_i}: *The position center of the mass*



Additional content:

Rotation vector:

$\vec{v} = \vec{\omega} \times \vec{r}$

$\|\vec{v}\| = \|\vec{w}\|\|\vec{r}\| \sin 90 = wr$